NATIONAL UNIVERSITY OF SINGAPORE

RESIT EXAMINATION FOR THE DEGREE OF B.ENG

(Semester II: 1996/97)

ME4245/EE4304 ROBOTICS

May 1997 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains four (4) questions, and comprises five (5) pages.
2. Answer all questions in the paper.
3. All questions carry equal marks.
4. This is an open-book examination.

Q1 (a) The relative position and orientation of two frames (A and C) are indicated in Fig. 1. Determine the homogeneous transformation matrices ${}^{C}T_{A}$, and ${}^{A}T_{C}$.

(6 marks)

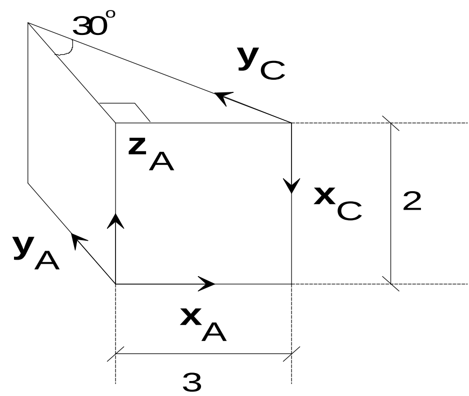

Fig. 1

(b) Two frames (B, and C) are fixed in space. The relative position and orientation of the frames are known. A third frame D is initially coincident to B. Frame D then undergoes the following rigid motion in the indicated sequence:

1st Rotation about z_B by 30°.
2nd Rotation about y_D by 45°.
3rd Rotation about x_B by 90°.
4th Rotation about x_C by 60°.

Determine an expression for the new position and orientation of Frame D in B after the four motions. (Express this in terms of known quantities; you do not need to numerically evaluate this.)

(13 marks)

(c) Complete the frame assignment for Frame 3 in Fig. 2 according to the Denavit-Hartenberg convention. Identify the four Denavit-Hartenberg parameters that relate frames 2 and 3.

(6 marks)

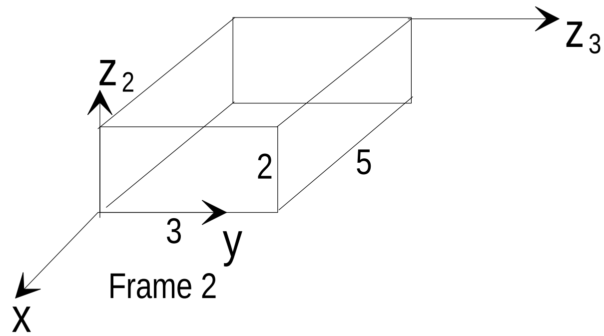

Fig. 2

Q2 Fig. 3 shows a schematic diagram of the 2-DOF robot. The first joint is a rotational joint whose axis of rotation is along the z-axis and its joint coordinate is θ. The second joint is a translational joint whose axis is along the second link's longitudinal axis.

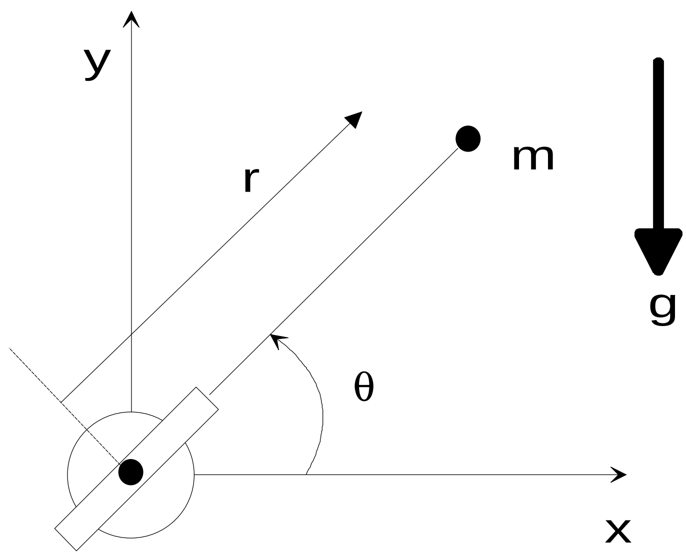

Fig. 3

(a) Derive an expression for the position of the point mass as a function of the joint coordinates.

(4 marks)

(b) Derive the **complete** inverse kinematic solutions for this robot. That is, derive the expressions for the two joint coordinates as functions of the position of the point mass.

(10 marks)

(c) Determine the 2x2 Jacobian for this robot.

(3 marks)

(d) Sketch the workspace of this robot.

(3 marks)

(e) Does this robot have any singularities? If so, determine the joint positions where the robot is at a singularity. How many degrees of freedom are lost at the singularity?

(5 marks)

Q.3 (a) Figure 3 shows a two-link (PR) planar robot in the vertical plane. Let m_i, i = 1, 2, be the lumped equivalent mass of link i, d be the translational displacement of link 1, q be the angular displacement of link 2, l be the length of link 2, f_1 be the external force on link 1 in the direction of d, and τ_2 be the torque to drive link 2 in the direction of q .

(i) What are the joint variables in the figure?

(2 marks)

(ii) Derive the Lagrange-Euler equations of the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \mathrm{t}$$

where $q = [q_1 \;\; q_2]^T$, $\mathrm{t} = [\mathrm{t}_1 \;\; \mathrm{t}_2]^T$, $D(q)$ is the inertia matrix, $C(q,\dot{q})$ is the matrix defined by the so-called Christoffel Symbols, and $G(q)$ is the gravitational forces.

(13 marks)

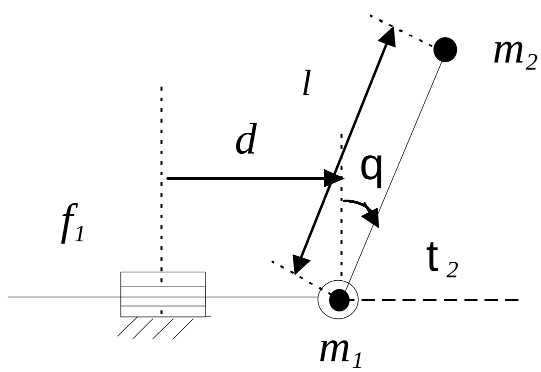

Figure 3: A PR robot in the vertical plane

(b) Give another different set of generalized coordinates for the robot shown in Figure 3.

(2 marks)

(c) Design a computed torque controller for this robot such that the resulting closed-loop system is decoupled, critically damped, and with natural frequency W = 5 rad/s. Discuss the advantages and disadvantages of the control scheme.

(8 marks)

Q.4 (a) For a single-link robot with a rotary joint, assume that the robot is motionless at $\theta = -10^{o}$. It is desired to move the joint in a smooth manner to $\theta = 80^{o}$ in 5 seconds. Find the coefficients of a cubic which accomplishes this motion and brings the arm to rest at the goal. Sketch profiles of position, velocity and acceleration.

(5 marks)

(b) How many individual cubics are computed when a five-jointed robot moves along a cubic spline path through three via points and stops at a goal point? How many coefficients are stored to describe these cubics?

(5 marks)

(c) A single cubic trajectory is given by

$$\theta(t) = 10 + 90t^{2} - 60t^{3}$$

and is used over the time interval from $t = 0$ to $t = 1$. What are the starting and final positions, velocities, and accelerations?

(5 marks)

(d) In path planning, list all the problems that should be avoided and explain.

(5 marks)

(e) Consider a system described by the following dynamic equations:

$$(m_1 l_1^2 + I_1 + m_2 d_2^2)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + 10(m_1 l_1 + m_2 d_2) cos\theta_1 = \tau_1$$

$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + 10 m_2 sin\theta_1 = f_2$$

where τ_1 and f_2 are the generalized torque and force for the first and the second degrees of freedom.

Determine the generalized coordinates, the inertia matrix, the coriolis force and centrifugal force vector, and the gravitational force vector. Are the equations correct? Why?

(5 marks)

END of PAPER