Adaptive Maintenance Policies for Ageing Devices Using a Markov Decision Process

Saranga K. Abeygunawardane, *Student Member, IEEE*, Panida Jirutitijaroen, *Member, IEEE*, and Huan Xu

***Abstract*— In competitive environments, most equipment are operated closer to or at their limits and as a result, equipment's maintenance schedules may be affected by system conditions. In this paper, we propose a Markov decision process (MDP) that allows better flexibility in conducting maintenance. The proposed MDP model is based on a state transition diagram where inspection and maintenance (I & M) delay times are explicitly incorporated. The model can be solved efficiently to determine adaptive maintenance policies. This formulation successfully combines the long term ageing process with more frequently observed short term changes in equipment's condition. We demonstrate the applicability of the proposed model using I & M data of local transformers.**

***Index Terms*-- asset management, condition monitoring (CM), maintenance, Markov decision processes, backward induction and transformers.**

I. NOMENCLATURE

S	The set of states
n	The total number of states
a	An action
t	The t^{th} decision epoch
i	The present state of the equipment
k	State at the decision epoch t+1
$P_t(k\|i, a)$	Probability of transiting from state i to any state $k \epsilon S$, upon choosing action a in state i at the t^{th} decision epoch
$r_t(i,a)$	Immediate reward for choosing action a in state i at the decision epoch t
$r_N(i)$	Boundary value of state i
C_i	The last known condition of the equipment
$t_{I,i}$	Time to perform next inspection when the last known condition is C_i (inspection delay time in C_i)
$t_{min,i}$	The minimum time between two consecutive inspections in C_i
$t_{max,i}$	The maximum allowable time between two consecutive inspections in C_i
τ	The interval at which I & M decision making is performed
N	The number of decision epochs
$n_{I,i}$	Number of consecutive times that the inspection is postponed
T	The decision horizon
a_0	Doing nothing
a_1	Inspection/ CM
a_2	Minor maintenance
a_3	Major maintenance
a_4	Replacement
a_5	Repair
$t_{M,i}$	Time spent in C_i (maintenance delay time in C_i)
$n_{max,i}$	The maximum number of decision intervals that the equipment spends in C_i
t_i	The maximum time period spent in condition C_i
$U_t(i,a)$	The total expected reward received upon choosing action a in state i at time t
$U_N^*(i)$	The maximum total expected reward in state i, at the N^{th} epoch
$U_{t+1}^*(k)$	The maximum total expected reward in state k, at the decision epoch t+1
$U_t^*(i)$	The maximum total expected reward in state i, at the t^{th} epoch
$a_t^*(i)$	The optimal action in state i at the decision epoch t

This work is supported by Singapore Ministry of Education-Academic Research Fund, Grant No. WBS R-263-000-691-112 and by Singapore Ministry of Education-National University of Singapore startup Grant R-265-000-384-133.

S. K. Abeygunawardane and P. Jirutitijaroen are with the Department of Electrical & Computer Engineering, National University of Singapore, Singapore (e-mail: g0800437@nus.edu.sg; elejp@nus.edu.sg).

H. Xu is with the Department of Mechanical Engineering, National University of Singapore, Singapore (e-mail: mpexuh@nus.edu.sg).

II. INTRODUCTION

ASSET management is essential for reliable and economic operation of power systems. With deregulation, asset management procedures became more complicated [1]. In such environments, an asset owner can perform preventive maintenance only after the independent system operator schedules a planned outage upon the request of the asset owner. In some situations, the operator may delay certain requested outages, in order to fulfill the overall aim of serving the power consumers [1]. Such situations require equipment owners to adjust their asset management plans accordingly. This highlights the need for adaptive asset management policies which can deal with maintenance delays.

Adaptive asset management policies would also be more economical than fixed policies. When the equipment is new and in good condition, too frequent inspection or CM would not reveal any additional information about the equipment's condition, and thus, unnecessarily increases the operation cost. On the other hand, when the equipment is aged or its condition is more deteriorated, delaying I & M may cause huge economic losses through unexpected failures. Hence, it would be more economical to perform I & M, considering the equipment's age, condition and delay times in I & M.

In literature, several mathematical models have been proposed to determine better maintenance policies for ageing assets [2-23]. However, there are some issues which are still not addressed in these previously proposed maintenance models. First, time delays in I & M are not included in most previous models [2-21]. Since optimal I & M actions may depend on I & M delay times, these delays should be considered when determining optimal policies.

Secondly, time based maintenance models represent equipment's deterioration by the overall condition based on the age [2-15], while condition based maintenance models represent the deterioration of the equipment by some observable measurements [16-23]. However, the deterioration of the equipment's measureable conditions may get accelerated with the ageing. Thus, it is more accurate if models can integrate the deterioration of equipment's measurable conditions with effects of ageing on deterioration. If a model can address the two aforementioned issues, such a model would be able to provide more adaptive I & M policies.

In this paper, we propose a new maintenance optimization model for I & M of ageing equipment. The proposed decision making model additionally considers delay times in performing I & M. Moreover, this model represents the deterioration of equipment using a quantifiable condition, while allowing the parameters of the deterioration process to vary with the operational age. Due to the above features of the model, it can provide more adaptive I & M policies which allow the asset owners to choose the optimal action, based on the knowledge about the equipment's *condition*, the operational *age* and *time delays* in performing I & M.

The structure of this paper is as follows. In section III, we provide the background theories and information. In section IV, the formulation of the maintenance optimization model is presented. In section V, the solution procedure is explained. A case study is presented in section VI to demonstrate a model application. Finally, conclusion is given in section VII.

III. Background

In this section, we first review the framework of a finite horizon discrete time MDP, with reference to [24]. Then, we discuss the decision making process regarding I & M of equipment. Lastly, we describe how this I & M decision making process is modeled in the framework of a finite horizon discrete time MDP.

A. *The Framework of a Markov Decision Process*

A MDP is a sequential decision making model which considers uncertainties in outcomes of current and future decision making opportunities. At each decision making time, the system/equipment occupies a state. Based on this state, a decision is made on choosing an action from the set of actions associated with this state. Upon choosing an action, a reward is received and a state transition occurs from the present state to a new state which is determined by a transition probability distribution. Since the process holds the Markov property, both transition probabilities and rewards only depend on the present state and the action chosen in the present state. As the process evolves, the decision maker receives a sequence of rewards. When choosing actions, the decision maker intends to maximize the total expected reward received over the total decision making period. If the total decision making period of a MDP is finite and the decisions are made in discrete time, the MDP is called a finite horizon discrete time MDP.

In standard practice, the decisions regarding I & M of equipment are made in discrete time. In addition, no equipment can be used over an infinitely long period and therefore, decisions regarding I & M of equipment are made over a finite time horizon. Due to these reasons, we represent the decision making process of equipment's I & M using a finite horizon discrete time MDP.

The five basic components of a finite horizon discrete time MDP are as follows.

1) Decision epochs: Decision epochs are the point of times at which decisions are made. In a discrete time MDP, the total decision making period (decision horizon) is divided into intervals which are called decision intervals, and at the beginning of each decision interval, a decision epoch occurs. The set of decision epochs is given by D={1, 2, 3, …, N}. In a finite horizon MDP, N is finite and according to the convention, decisions are not made at the Nth decision epoch. Decision horizon, decision intervals and decision epochs of a discrete time finite horizon MDP are shown in Fig. 1.

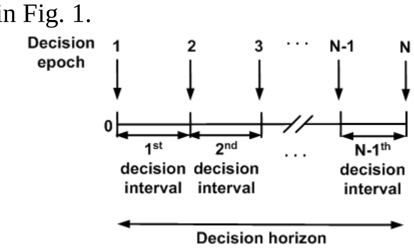

Fig. 1. Decision horizon, decision intervals and decision epochs.

2) States: Different statuses of a system/equipment are modeled using a finite number of states.
3) Actions: Each state is connected with a finite number of actions.
4) Transition probabilities: As a result of choosing any action a connected with state i at the t^{th} decision epoch, a state transition occurs. The new state at the decision epoch t+1 is determined by the probabilities of transiting from state i to possible states in the state space S. The probability of transiting from state i to any state $k \in$ S, upon choosing action a in state i at the t^{th} decision epoch is denoted by $\mathrm{P}_t(k|i, a)$. It should be noted that $\sum_{k=1}^{n} \mathrm{P}_t(k|i, a) = 1$.
5) Rewards: At each decision epoch t <N, the decision maker receives a reward, as a result of choosing an action. The reward received upon choosing action a in state i at the t^{th} decision epoch is denoted by $\mathrm{r}_t(i, a)$. The reward received at the Nth decision epoch is assigned based on the state that the equipment is being found at the Nth decision epoch. This is called the boundary value, and the boundary value of state i is denoted by $\mathrm{r_N}(i)$.

A MDP can be symbolically represented using a state transition diagram. A simple state transition diagram is given in Fig. 2 to provide a better understanding about some of the aforementioned basic components. This model in Fig. 2 has two states namely S_1 and S_2, and three actions namely a_1, a_2 and a_3. The state S_1 is connected with actions a_1 and a_3, while the state S_2 is connected with actions a_2 and a_3. Rewards and transition probabilities associated with actions and states at any decision epoch t <N are also given in Fig. 2.

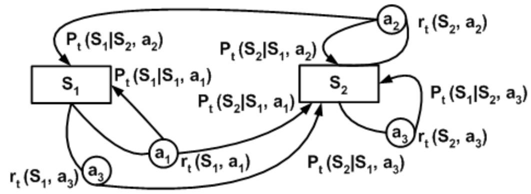

Fig. 2. The simple state transition diagram of a MDP model.

B. Inspection and Maintenance Decision Making in Actual Practice

In practice, the condition of equipment is assessed through online or offline inspections. These inspections are usually performed at scheduled intervals as specified in the standards or by the manufacturer. Based on the results of inspections, the condition of equipment can be interpreted using one of the finite number of deterioration stages i.e. C_1, C_2, …, C_j, where C_1 and C_j represents the best and the worst conditions, respectively. In order to improve the present condition C_i of the equipment, maintenance is performed. If no maintenance is performed, the condition gradually deteriorates from C_i to C_j. However, maintenance may also degrade the present condition C_i or may not change C_i. When the equipment is at any deterioration stage, there is a certain probability of failure, which usually increases with deterioration.

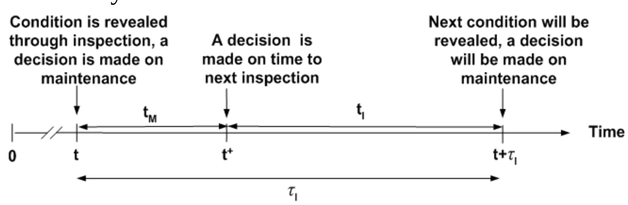

Fig. 3. Decision making process regarding inspection and maintenance.

Two consecutive decisions are made repeatedly throughout the equipment's operational life; one on maintenance and the other on time to next inspection. As shown in Fig. 3, when the condition is revealed through inspection at time t, the first decision is made regarding the required maintenance action. Followed by this decision, the second decision is made at time t^+ regarding the time to perform next inspection i.e. t_I. In Fig. 3, t_M denotes the time taken to perform maintenance, and τ_I is the time interval between two consecutive inspections. Since $t_M << t_I$, whether maintenance is performed or not, the time gap between t and t^+ is considered small and thus, $\tau_I \approx t_I$.

Generally, asset owners tend to decrease the time to next inspection with the deterioration of the equipment's condition. Therefore, the value of t_I depends on the equipment's condition C_i. We denote this time interval corresponding to the last known condition C_i by $t_{I,i}$. In order to achieve more cost effective I & M policies, it is possible to vary $t_{I,i}$ within a range $t_{min,i} \leq t_{I,i} \leq t_{max,i}$, depending on other considerations such as the equipment's age and time delays in performing I & M. $t_{min,i}$ and $t_{max,i}$ refer to the minimum and the maximum allowable time between two consecutive inspections in stage C_i. These parameters can be determined based on inspection histories and experts' opinion.

C. Modeling the Process of Decision Making

In the MDP framework, it is required that the decisions are made at constant intervals. When modeling the decision making process of I & M, we first determine a common time slot (τ) to perform decision making on I & M. It would be more accurate to choose a small duration for τ and let the model to make decisions regarding I & M in each τ. However, in practice, when the equipment is at good condition, inspections are performed at a lower frequency. A large data set is required to accurately calculate deterioration probabilities for each interval τ, especially when the equipment's deterioration condition is good. As shown in (1), τ is set to the greatest common divisor of all $t_{min,i}$ values suggested by industry experts.

$$\tau = \gcd\left(t_{min,1}, t_{min,2}, \cdots, t_{min,j}\right) \quad (1)$$

Within the interval τ, two decisions are made. The first decision is regarding maintenance activities, whereas the second decision is regarding the next inspection. Decision making regarding I & M within each interval τ is modeled similar to the actual situation shown in Fig. 3. That is, if a decision is made regarding maintenance at time t and if the time taken to implement this decision is t_M, the decision regarding the next inspection is made at time t^+ (or t+t_M). Then, the time to next decision making on maintenance is τ-t_M. It is worth to note that MDP theory does not require t_M (the time from the maintenance decision to the inspection decision) and τ-t_M (the time from the inspection decision to the maintenance decision) to be equal. However, it is required to keep these time intervals consistent for different I & M trajectories. In the proposed MDP model, t_M is the same for each I & M trajectory. Since τ is kept constant for each I & M trajectory, τ-t_M is also the same for each trajectory. Moreover, in each and every possible trajectory, the decision maker makes the maintenance decisions on odd decision stages only and the decisions of next inspection on even decision stages only. Thus, the time intervals t_M and (τ-t_M) are kept consistent for different I & M trajectories.

In the MDP framework, the second decision is not regarding time to next inspection, but regarding whether to perform the next inspection at the end of the current interval τ or to wait till the next interval τ to make a decision on the next inspection. However, time to next inspection ($t_{I,i}$) is related to the number of the next consecutive intervals of τ in which the inspection is postponed ($n_{I,i}$). For example, assume that inspection was performed in the previous interval τ. If it is decided to perform inspection again at the end of the current interval τ (i.e. if inspection is not postponed to the next interval τ), $n_{I,i}$ is zero and $t_{I,i}$ is τ. If the next inspection is postponed by $n_{I,i}$ consecutive times, $t_{I,i}$ is given using (2).

$$t_{I,i} = \tau \times (n_{I,i} + 1) \quad (2)$$

Although the decision making interval regarding I & M is considered to be τ, I & M can be practically performed only once in each $t_{min,i}$. Thus, the MDP model should allow choosing I & M actions only once in each $t_{min,i}$. When $t_{min,i} > \tau$, this property is incorporated into the model by eliminating I & M actions connected with some states of the equipment. We will explain this further in section IV (B), using the state transition diagram of the proposed MDP model.

IV. Problem Formulation

In this section, we describe the concept of the proposed MDP model for I & M decision making of ageing equipment. In subsections *A*-*C*, the components of the proposed MDP model are described using the state diagram of the proposed model in Fig. 4. For this description, we consider a simplified model developed for equipment having two deterioration stages, C_1 and C_2 with $t_{min,1}$, $t_{max,1}$ $t_{min,2}$, and $t_{max,2}$ of 3τ, 6τ, τ, and 2τ, respectively. However, this proposed model can be applied to equipment with any number of deterioration stages. In subsection *D*, we discuss how the proposed MDP model combines the effect of ageing with the deterioration process of the equipment's measurable condition.

A. Decision epochs

These are the point of times at which decisions are made regarding equipment's inspection or maintenance. The number of decision epochs (N) of the proposed MDP is given by (3), where T is the decision horizon. Since I & M decisions are made throughout the equipment's total operational period, we set the expected operational life of the equipment for T. T/τ in this equation gives the number of I & M decision making intervals in the decision horizon T. It should be noted that two decision epochs occur within each I & M decision making interval τ. If the decision epoch is odd, decisions are made only regarding inspection. Otherwise, decisions are made only regarding maintenance.

$$N=\left(\frac{T}{\tau}\right)\times 2+1 \tag{3}$$

B. States and Actions

Different statuses of the equipment at time points of I & M decision making are modeled using different states. In order to decouple decision making on inspection and decision making on maintenance, we define two types of equipment states, namely main states and intermediate states. This decoupling is essential, to model the practical scenario, where a decision is made on inspection prior to decision making on maintenance and the decision regarding maintenance is made depending on the outcomes of the inspection. At main states, decisions are made only regarding maintenance and at intermediate states, decisions are made only regarding the next inspection. (For example, with respect to Fig. 3, the possible states of the equipment at *t* and $t+\tau_I$ are called main states and the possible states at t^+ are called intermediate states.) These main and intermediate states of the proposed model are shown in Fig. 4 using solid and dashed rectangles, respectively. The set of actions includes doing nothing (a_0), inspection (a_1), minor maintenance (a_2), major maintenance (a_3), replacement (a_4) and repair (a_5).

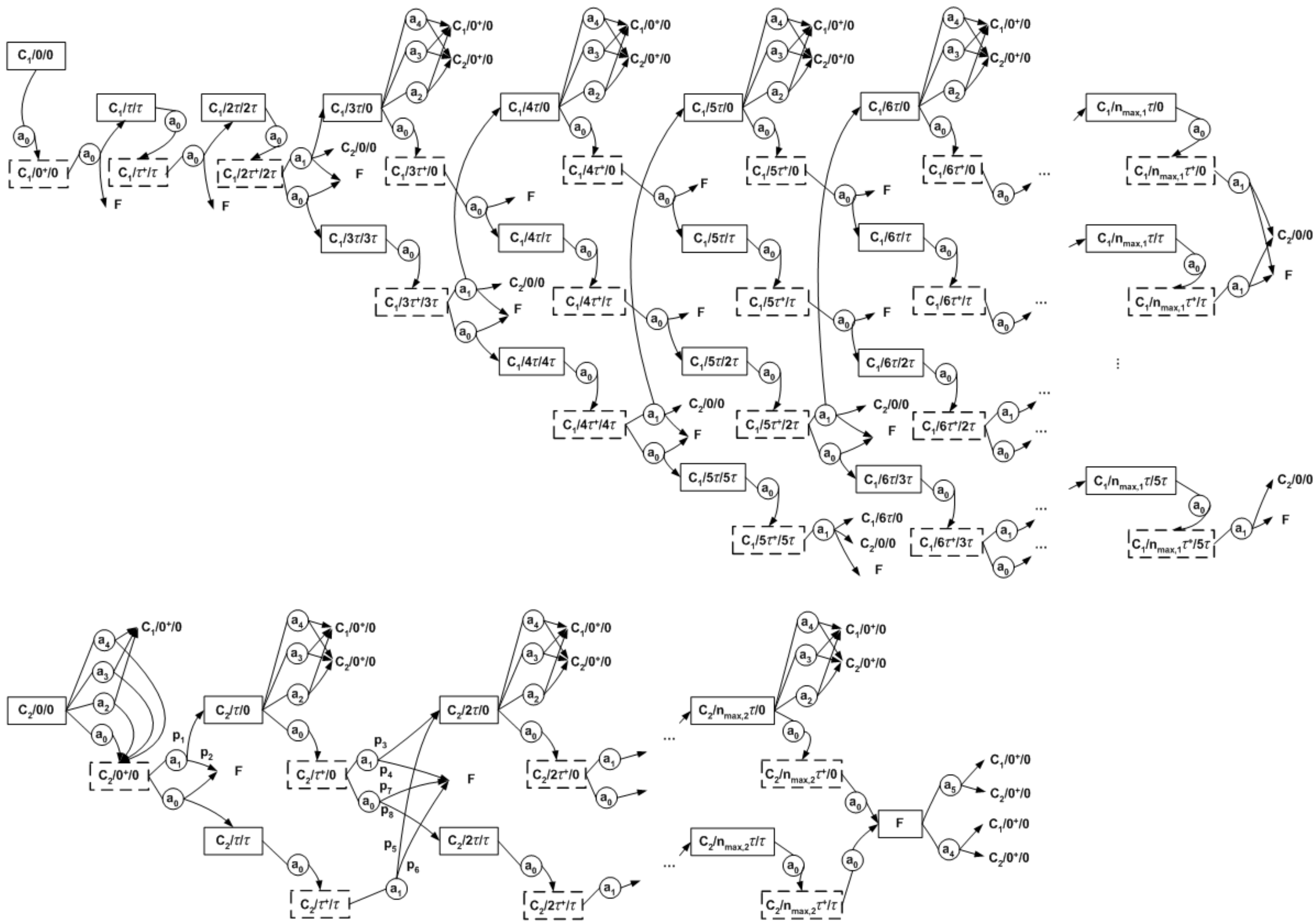

Fig. 4. The state transition diagram of the proposed MDP model for maintenance decision making.

In the MDP model, we describe a deterioration state by $C_i/t_{M,i}/t_{I,i}$. C_i denotes the deterioration stage of the equipment where i = 1, 2. $t_{M,i}$ is the time spent in C_i, which can also be called as the maintenance delay time in stage C_i. $t_{I,i}$ is the time from the most recent inspection i.e. the inspection delay time in stage C_i. According to this state convention, the status of newly installed equipment is represented by the state $C_1/0/0$. The failure state is denoted by F. This failure state F only stands for the deterioration failures of the equipment. Since random failures cannot be avoided by performing I & M activities, such failures are not considered in the model.

States are connected with associated actions, as shown in the state transition diagram in Fig. 4. This diagram also shows how possible state transitions occur upon choosing each action at each state. Since the decisions made at intermediate states are about postponing the next inspection, intermediate states are connected only with actions a_0 and a_1. If the equipment does not fail (i.e. if the next state is not the state F), the two actions a_0 and a_1 lead to different main states. If action a_0 is chosen at an intermediate state, these next possible main states are connected only with action a_0. If action a_1 is chosen at an intermediate state, next possible main states are connected with actions a_0, a_2, a_3 and a_4. However, the following exceptions can be noted in Fig. 4.

- When equipment is newly installed, it is not required to perform maintenance, replacement or repair. Therefore, the main state $C_1/0/0$ is only connected with action a_0.
- Once the equipment fails, it must be replaced or repaired and hence the main state F is connected with a_4 and a_5.
- The minimum possible time between two consecutive inspections in stage C_1, $t_{min,1}$ is 3τ. Due to this reason, when the condition is C_1, the model permits to choose inspection only if the inspection delay time $t_{I,1} \geq 2\tau$. Otherwise, inspection is not allowed and therefore, action a_1 is not connected to the grey colored intermediate states in Fig. 4.
- The maximum time that the inspection can be delayed at C_1 and C_2 (i.e. $t_{max,1}$ and $t_{max,2}$) are 6τ and 2τ, respectively. Therefore, decisions must be made to perform inspection, when $t_{I,1}$ is 5τ or $t_{I,2}$ is τ, at an intermediate state. As Fig. 4 shows, such intermediate states are connected only with a_1.
- There is a maximum time period that the equipment spends in each condition, before it deteriorates further or fails. This maximum time period spent in condition C_i (t_i) can be determined from inspection and failure history. With the use of t_i, the maximum number of decision intervals that the equipment spends in C_i ($n_{max,i}$) can be determined as shown in (4). MDP model assumes that inspections must be performed, when the time spent in stage C_i is $n_{max,i}\tau$. Thus, if $t_{M,i}$ of an intermediate state is $n_{max,i}\tau$, decisions are made to perform inspection. As can be seen in Fig. 4, such intermediate states are connected only with a_1. However, if C_i is the last deterioration stage, at the end of the interval $n_{max,i}\tau$, the equipment fails whether inspection is performed or not. Therefore, as shown in Fig. 4, corresponding intermediate states are connected only with action a_0.

$$n_{max,i} = \frac{t_i}{\tau} - 1 \tag{4}$$

C. Transition Probabilities and Rewards

Using some notations (i.e. transition probabilities p_1-p_8) given in Fig. 4, we illustrate how transition probabilities are calculated for the proposed MDP model. Transition probabilities corresponding to action a_1 are the deterioration/failure probabilities of the equipment. Deterioration/failure probabilities associated with zero inspection delay time (e.g. p_1-p_4 in Fig. 4) can be directly calculated using inspection and failure history. For example, let us denote the number of transformers found to be in condition C_2 for a period of τ by n_1, the number of transformers found to be in condition C_2 for a period of 2τ by n_2. From this data, $p_3 = n_2/n_1$ and $p_4 = (n_1 - n_2)/n_1$.

When the inspection delay time is greater than zero, deterioration/failure probabilities can be calculated with the use of deterioration/failure probabilities corresponding to zero inspection delay time. For example, consider the calculation of p_5 and p_6, which are the probabilities of being found in C_2 for a period of 2τ and being found in the failure state F, if the inspection is delayed by an interval τ. Let the events,

$$A = \{\text{Equipment being found in } C_2 \text{ for a period } \tau\}$$
$$B = \{\text{Equipment being in } C_2 \text{ for a period } 2\tau\}$$
$$B|A = \{\text{Equipment being found in } C_2 \text{ for a period } 2\tau \,|\, \text{Equipment being found in } C_2 \text{ for a period } \tau\}$$

Using the conditional probability rule,

$$P(B|A)=P(A\cap B)/P(A)=P(B)/P(A)$$
$$\therefore P(B)=P(A)\times P(B|A)$$

By substituting values,

$$p_5 = p_1 \times p_3 \tag{5}$$

Similarly, p_6 can also be computed as follows.

$$p_6 = p_1 \times p_4 + p_2 \tag{6}$$

Since failures can occur whether inspection is performed or not, failure probability is the same for both actions a_0 and a_1. When the condition is C_i, if the equipment does not fail upon choosing action a_0, the model assumes that the condition would remain same as C_i. Thus, probabilities corresponding to a_0 can be simply calculated using the failure probabilities calculated for a_1. For example, $p_7 = p_4$ and $p_8 = 1 - p_7$.

Transition probabilities corresponding to maintenance and repair actions (i.e. a_2, a_3 and a_5) can be calculated using maintenance/repair records and the above method of calculation which is used to find deterioration probabilities. When equipment is replaced (i.e. action a_4 is chosen), the probability of transiting to state $C_1/0^+/0$ is considered to be 1.

For each action a reward is allocated equal to the negative of the cost of performing that particular action. Boundary value of each state is set to zero, assuming that the value of the equipment at the end of the expected operational life is zero.

D. Incorporating the Effects of Aging

Deterioration of equipment's condition generally gets accelerated with the ageing. In addition, when equipment is aged, it would be less possible to improve the condition through maintenance. These effects of ageing on deterioration of equipment's measurable condition should be reflected in I & M history. Thus, deterioration probabilities and the probabilities of improving the condition after maintenance/repair would be different for different age levels of the equipment. Since the transition probabilities (i.e. $\mathrm{P}_t(k|i, a)$) of a MDP model can vary with time t, the proposed MDP model can easily incorporate the effects of ageing.

Incorporating the effects of ageing can be done as follows, given that data is available over the equipment's expected life. First, the expected life is divided into an appropriate number of age levels (j), as shown in Fig. 5. Then, I & M data collected over the expected life of the equipment is categorized into different groups corresponding to these age levels. Next, the classified data is used to compute deterioration probabilities and maintenance/repair outcome probabilities for each age level. These probabilities calculated for a particular age level are set for transition probabilities of the decision epochs which belong to that particular age level. For example, with reference to Fig. 5, the probabilities calculated using the data corresponding to the 2nd age level are set for all $\mathrm{P}_t(k|i, a)$, where $t \in \{x, x+1, x+2, \ldots, y\}$.

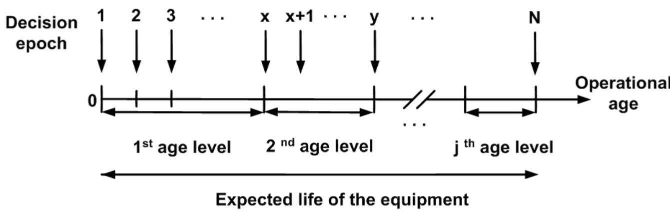

Fig. 5. Decision epochs at different age levels of the equipment.

V. Solution Procedure

The combinations of states and actions of our proposed MDP model result in a large number of possible maintenance policies. We use backward induction i.e. dynamic programming to solve the MDP efficiently [24]. This technique can provide the optimal policies without analyzing every possible policy. In backward induction, the final stage of decision making at t=N is first attended and the decisions on optimal actions are made by moving one step backward at each decision epoch in the desired time horizon. Intuitively, backward induction works because an action of an intermediate state s is optimal only if it is optimal for a reduced MDP starting from s [24].

When solving a MDP, the objective is to decide the optimal set of actions which maximizes the total expected reward. In backward induction method, when t=N, for any state i, the maximum total expected reward $\mathrm{U}^*_{\mathrm{N}}(i)$ is set to the boundary value of state i. Then, when t<N, the maximum total expected reward for state i at time t, (i.e. $\mathrm{U}^*_t(i)$) is found as follows.

- Using (7), $\mathrm{U}_t(i,a)$ i.e. the total expected reward received upon choosing action a in state i at time t is calculated. In (7), $\mathrm{r}_t(i,a)$ is the immediate reward received upon choosing action a, which is basically the reward assigned for action a in state i at time t. The term $\sum_{k=1}^{\mathrm{n}} \mathrm{P}_t(k|i, a) \times \mathrm{U}^*_{t+1}(k)$ is the expected terminal reward, where n is the total number of states, $\mathrm{P}_t(k|i, a)$ is the probability of transiting to state k, if action a is chosen in state i at the epoch t and $\mathrm{U}^*_{t+1}(k)$ is the maximum total expected reward in state k, at the epoch t+1.

$$\mathrm{U}_t(i,a)=\mathrm{r}_t(i,a)+\sum_{k=1}^{\mathrm{n}}\left\{\mathrm{P}_t(k|i, a)\times\mathrm{U}^*_{t+1}(k)\right\} \tag{7}$$

- Once the total expected reward is found for every possible action in state i, the maximum total expected reward in state i, at the tth epoch is found using the criterion in (8).

$$\mathrm{U}^*_t(i) = \max\{\mathrm{U}_t(i,a)\} \tag{8}$$

The optimal action in state i at the decision epoch t can be obtained using (9).

$$a^*_t(i) = \arg\max\{\mathrm{U}_t(i,a)\} \tag{9}$$

Likewise, at each decision epoch t, an optimal action which maximizes the expected total reward can be found for all relevant states. As mentioned before in section IV (A), at odd decision epochs optimal actions are found for all intermediate states where decisions are made regarding inspection. At even epochs, optimal actions are found for all main states, where decisions are made regarding maintenance. The set of optimal actions of all relevant states at the decision epoch t is called the solution of MDP at time t.

To find optimal maintenance policies by solving our proposed MDP model, we implement the following backward induction algorithm in MATLAB 10.

Step 1: Set t=N and $\mathrm{U}^*_{\mathrm{N}}(i) = \mathrm{r}_{\mathrm{N}}(i)$ for all $i \in$(1,n)
Step 2: Set $t = t$-1
Step 3: Set i =1
Step 4: Continue only if, t is odd and i is an intermediate state, or t is even and i is a main state. Otherwise, it is not required to find $a^*_t(i)$ and thus, skip steps 5 and 6 and set $\mathrm{U}^*_t(i)$=$\mathrm{U}^*_{t+1}(i)$.
Step 5: Compute $\mathrm{U}_t(i,a)$ for each action a available in state i using (7).
Step 6: Find the optimal action for state i, using (8) and (9).
Step 7: If i=n, stop. Otherwise, set i=i+1 and go to step 4.
Step 8: If t=1, stop. Otherwise repeat from step 2.

VI. Case Study

In this section, we present a case study in which the proposed MDP model is applied to CBM of oil insulated distribution transformers.

A. CBM of Oil Insulated Transformers

Utilities basically assess the condition of oil insulated transformers through dissolved gas analysis (DGA) [25, 26]. In DGA, insulation oil is sampled at scheduled intervals, while the transformer is in operation, and the amounts of dissolved gases are measured and analyzed. Then, the condition is determined using the total amount of dissolved combustible gases (TDCG) according to the criterion specified in the IEEE standards [27]. Next, based on the revealed condition, maintenance decisions are made considering recommendations in the standards [27].

B. The MDP Model of Transformers

The data required for the MDP model of transformers include DGA results, maintenance, repair and replacement records of transformers and costs of performing CM, maintenance, repair and replacement actions. DGA results are only available over past 7 years, as DGA is recently introduced for distribution transformers in the local utility. However, in order to demonstrate the model applicability, we conduct the case study with these DGA results and maintenance records which belong to the transformers' age range of 20 to 30 years. We assume CM, maintenance and replacement costs based on [6]. Using this data and considering current CM and maintenance practices and experts' opinion, model parameters are determined.

TABLE I
DETERIORATION/FAILURE PROBABILITIES.

C_i	$t_{M,i}$ / (years)	Probability of transition to condition C_1, C_2, C_3 or F											
		0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
		C_1	C_2	C_3	F	C_1	C_2	C_3	F	C_1	C_2	C_3	F
C_1	0	1	0	0	0	1	0	0	0	1	0	0	0
	0.33	1	0	0	0	1	0	0	0	1	0	0	0
	0.67	1	0	0	0	1	0	0	0	1	0	0	0
	1	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	1.33	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	1.67	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	2	1	0	0	0	0.94	0.06	0	0	0.90	0.10	0	0
	2.33	1	0	0	0	0.94	0.06	0	0	0.89	0.11	0	0
	2.67	1	0	0	0	0.94	0.06	0	0	0.88	0.12	0	0
	3	1	0	0	0	0.90	0.10	0	0	0.67	0.33	0	0
	3.33	1	0	0	0	0.89	0.11	0	0	0.50	0.50	0	0
	3.67	1	0	0	0	0.88	0.12	0	0	0	1	0	0
	4	0.94	0.06	0	0	0.67	0.33	0	0	-	-	-	-
	4.33	0.94	0.06	0	0	0.50	0.50	0	0	-	-	-	-
	4.67	0.94	0.06	0	0	0	1	0	0	-	-	-	-
	5	0.90	0.10	0	0	-	-	-	-	-	-	-	-
	5.33	0.89	0.11	0	0	-	-	-	-	-	-	-	-
	5.67	0.88	0.12	0	0	-	-	-	-	-	-	-	-
	6	0.67	0.33	0	0	-	-	-	-	-	-	-	-
	6.33	0.50	0.50	0	0	-	-	-	-	-	-	-	-
	6.67	0	1	0	0	-	-	-	-	-	-	-	-
C_2	0	0	1	0	0	0	1	0	0	0	1	0	0
	0.33	0	1	0	0	0	1	0	0	0	0.89	0.11	0
	0.67	0	1	0	0	0	1	0	0	0	0.75	0.25	0
	1	0	1	0	0	0	0.89	0.11	0	0	0.67	0.33	0
	1.33	0	1	0	0	0	0.75	0.25	0	0	0	1	0
	1.67	0	1	0	0	0	0.67	0.33	0	-	-	-	-
	2	0	0.89	0.11	0	0	0	1	0	-	-	-	-
	2.33	0	0.75	0.25	0	-	-	-	-	-	-	-	-
	2.67	0	0.67	0.33	0	-	-	-	-	-	-	-	-
	3	0	0	1	0	-	-	-	-	-	-	-	-
C_3	0	0	0	1	0	0	0	1	0	0	0	1	0
	0.33	0	0	1	0	0	0	1	0	0	0	0.8	0.2
	0.67	0	0	1	0	0	0	0.8	0.2	0	0	0.6	0.4
	1	0	0	0.8	0.2	0	0	0.6	0.4	0	0	0	1
	1.33	0	0	0.6	0.4	0	0	0	1	-	-	-	-
	1.67	0	0	0	1	-	-	-	-	-	-	-	-

TABLE II
TRANSITION PROBABILITIES UPON CHOOSING MAINTENANCE ACTIONS AT C_3.

Action	$t_{M,i}$ / (years)	Probability of transition from C_3 to other conditions											
		0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
		C_1	C_2	C_3	F	C_1	C_2	C_3	F	C_1	C_2	C_3	F
a_2	0	0	1	0	0	0	1	0	0	0	0.7	0.3	0
	0.33	0	1	0	0	0	0.7	0.3	0	0	0.4	0.6	0
	0.67	0	0.7	0.3	0	0	0.4	0.6	0	0	0.2	0.8	0
	1	0	0.4	0.6	0	0	0.2	0.8	0	0	0	0.5	0.5
	1.33	0	0.2	0.8	0	0	0	0.5	0.5	-	-	-	-
	1.67	0	0	0.5	0.5	-	-	-	-	-	-	-	-
a_3	0	1	0	0	0	1	0	0	0	0.9	0.1	0	0
	0.33	1	0	0	0	0.9	0.1	0	0	0.8	0.2	0	0
	0.67	0.9	0.1	0	0	0.8	0.2	0	0	0.6	0.4	0	0
	1	0.8	0.2	0	0	0.6	0.4	0	0	0.5	0.5	0	0
	1.33	0.6	0.4	0	0	0.5	0.5	0	0	-	-	-	-
	1.67	0.5	0.5	0	0	-	-	-	-	-	-	-	-

Based on TDCG, IEEE standards specify four deterioration conditions of transformers [27]. We model them as three conditions i.e. by separately considering the first two conditions and by combining the third and fourth conditions. According to the degree of deterioration, we denote the three conditions by C_1, C_2 and C_3. The minimum and maximum CM intervals, $t_{min,1}$, $t_{max,1}$, $t_{min,2}$, $t_{max,2}$, $t_{min,3}$, and $t_{max,3}$ in years are 1, 3, 0.33, 1.33, 0.33 and 1, respectively. Using (1), I &M decision making interval is chosen as 0.33 years. Data shows that the maximum time spent in C_1, C_2 and C_3 are 5, 2.33 and 1.67 years, and therefore, $n_{max,1}$, $n_{max,2}$ and $n_{max,3}$ are 14, 6 and 4, respectively. According to these model parameters, the state diagram consists of 370 states. The set of actions that are performed on local transformers includes doing nothing (a_0), CM (a_1), minor maintenance (a_2), major maintenance (a_3) and replacement (a_4), respectively.

Assuming that the total expected life of a transformer is 40 years, T is set to 40 years. Then, from (3), N = 241. In this case study, three sets of transition probabilities are utilized for three age levels, i.e. 0 to 20 years, 20 to 30 years and 30 to 40 years. Deterioration/failure probabilities for these three age levels are given in table I. For the age range of 20 to 30 years, these deterioration probabilities are computed using the available DGA results. As there are no occurrences of failures, failure probabilities are interpolated. Probabilities corresponding to the age range of 20 to 30 years are appropriately amended to obtain other deterioration/failure probabilities in table I which are corresponding to the other two age ranges. Using these deterioration probabilities in table I, transition probabilities corresponding to actions a_1 and a_0 are calculated. Maintenance history shows that, actions a_2, a_3 and a_4 are not performed, when the condition is C_1. By performing a_2 or a_3, the condition is improved from C_2 to C_1. Upon choosing a_2 or a_3 in C_3, transitions occur according to the probabilities given in table II. If action a_4 is chosen at any state, the condition is improved to C_1. Rewards assigned for actions a_0, a_1, a_2, a_3 and a_4 are 0, -200, -1200, -14400 and -144000, respectively [6]. Boundary values are set to zero.

The MDP model with these parameters is solved using the backward induction algorithm to find optimal actions.

C. *Results and Discussion*

Although it is possible to select optimal actions directly from the solution of the MDP, for easy reference, we convert the solution into look up tables given in tables III-VI.

Based on the current condition, the time spent in this condition, the CM delay time, and the operational age, the optimal decision regarding CM can be chosen from tables III-V. According to the model, these decisions are to be implemented at the end of the next 4 months. In tables III-V, CM actions which are pre-specified during the modeling of the state diagram are mentioned in bold. Apart from these pre-specified CM actions, the results in tables III-V suggest performing some additional CM. The overall implications of the results in tables III-V are explained below.

1) With the ageing of a transformer, the probability of deterioration and failure increases. Therefore, it is not cost effective to delay CM too much, if the equipment is old.
2) Similarly, when the equipment is more deteriorated, the equipment is at a higher risk of failure. Thus, it would be cost effective to perform CM with a less delay.
3) With the increase in time that the equipment spends in a condition, the probability of deterioration increases. If CM is delayed too much, the transformer may further

deteriorate unknown to the maintenance staff and require more maintenance to improve the condition or it may fail unexpectedly. Thus, it would be more cost effective to perform CM without any delay, if the time spent in a condition (or maintenance delay time) is high.

TABLE III
OPTIMAL ACTIONS TO PERFORM CM AT C_1.

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
0	0	a_0	a_0	a_0
0.33	0.33	a_0	a_0	a_0
0.67	0.67	a_0	a_0	a_0
1	0, 1	a_0	a_0	a_0
1.33	0, 0.33, 1.33	a_0	a_0	a_0
1.67	0, 0.33, 0.67, 1.67	a_0	a_0	a_0
2	0, 0.33, 0.67, 1, 2	a_0	a_0	a_0
2.33	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 2.33	a_0	a_0	a_1
2.67	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67	a_0	a_0	a_1
	2.67	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$
3	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67, 2	a_0	a_0	a_1
3.33	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67, 2, 2.33	a_0	a_0	a_1
3.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	a_0	$\mathbf{a_1}$
	2.67	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$
4	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	a_0	-
	2.67	$\mathbf{a_1}$	$\mathbf{a_1}$	-
4.33	0, 0.33, 0.67, 1, 1.33, 1.67, 2	a_0	a_0	-
	2.33	a_0	a_1	-
	2.67	$\mathbf{a_1}$	$\mathbf{a_1}$	-
4.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	$\mathbf{a_1}$	-
	2.67	$\mathbf{a_1}$	$\mathbf{a_1}$	-
5, 5.33, 5.67, 6, 6.33	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	-	-
	2.67	$\mathbf{a_1}$	-	-
6.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33, 2.67	$\mathbf{a_1}$	-	-

Based on the condition, the maintenance delay time, and the operational age of a transformer, the optimal decision regarding maintenance can be chosen from table VI. These maintenance decisions are for immediate implementation. Implications of the results in table VI are given below.

1) With the ageing of the equipment, the failure probability would increase and therefore, the time that the maintenance can be delayed decreases.
2) It is not cost effective to delay maintenance, when the equipment is more deteriorated and at a higher risk.
3) Cost effective maintenance actions would change with the maintenance delay time. For example, as the time spent in C_3 increases, the probability of improving the condition by performing minor maintenance decreases and thus, it will be more cost effective to perform major maintenance.

TABLE IV
OPTIMAL ACTIONS TO PERFORM CM AT C_2.

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
0	0	a_0	a_0	a_0
0.33	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_0
0.67	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_1
	0.67	a_0	a_0	a_1
1	0	a_0	a_0	a_1
	0.33	a_0	a_0	a_1
	0.67	a_0	a_0	a_1
	1	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$
1.33	0	a_0	a_0	$\mathbf{a_1}$
	0.33	a_0	a_0	$\mathbf{a_1}$
	0.67	a_0	a_0	$\mathbf{a_1}$
	1	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$
1.67	0	a_0	a_0	-
	0.33	a_0	a_0	-
	0.67	a_0	a_1	-
	1	$\mathbf{a_1}$	$\mathbf{a_1}$	-
2	0	a_0	$\mathbf{a_1}$	-
	0.33	a_0	$\mathbf{a_1}$	-
	0.67	a_0	$\mathbf{a_1}$	-
	1	$\mathbf{a_1}$	$\mathbf{a_1}$	-
2.33	0	a_0	-	-
	0.33	a_0	-	-
	0.67	a_0	-	-
	1	$\mathbf{a_1}$	-	-
2.67	0	a_0	-	-
	0.33	a_0	-	-
	0.67	a_0	-	-
	1	$\mathbf{a_1}$	-	-
3	0	$\mathbf{a_1}$	-	-
	0.33	$\mathbf{a_1}$	-	-
	0.67	$\mathbf{a_1}$	-	-
	1	$\mathbf{a_1}$	-	-

TABLE V
OPTIMAL ACTIONS TO PERFORM CM AT C_3.

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
0	0	a_0	a_0	a_1
0.33	0	a_0	a_1	a_1
	0.33	a_0	a_1	a_1
0.67	0	a_1	a_1	$\mathbf{a_1}$
	0.33	a_1	a_1	$\mathbf{a_1}$
	0.67	$\mathbf{a_1}$	$\mathbf{a_1}$	$\mathbf{a_1}$
1	0	a_1	$\mathbf{a_1}$	-
	0.33	a_1	$\mathbf{a_1}$	-
	0.67	$\mathbf{a_1}$	$\mathbf{a_1}$	-
1.33	0	$\mathbf{a_1}$	-	-
	0.33	$\mathbf{a_1}$	-	-
	0.67	$\mathbf{a_1}$	-	-

TABLE VI
OPTIMAL ACTIONS TO PERFORM MAINTENANCE.

Condition	$t_{M,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
C_2	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_0
	0.67	a_0	a_0	a_2*
	1	a_0	a_0	a_2
	1.33	a_0	a_0	a_2
	1.67	a_0	a_2	-
	2	a_0	a_2	-
	2.33	a_0	-	-
	2.67	a_0	-	-
	3	a_0	-	-
C_3	0	a_0	a_2	a_2
	0.33	a_0	a_2*	a_2
	0.67	a_2*	a_2	a_2
	1	a_2	a_2	a_3
	1.33	a_2	a_3	-
	1.67	a_3	-	-

Since CM must be performed before maintenance, some of the suggested maintenance actions are not implementable. Such actions in table VI are denoted using an additional "*". In order to guarantee that CM is performed before each maintenance activity, the equipment operators should first refer tables III-V for the optimal CM action, and only if tables III-V suggest performing CM they should then refer table VI for the optimal maintenance action.

VII. Conclusion

With the power system deregulation, asset owners would prefer to adopt more adaptive and cost effective maintenance policies. In this paper, we propose a maintenance optimization model based on a MDP to find such maintenance policies for ageing electrical equipment. Deterioration states of this proposed MDP model are more detailed. Thus, this model is capable of incorporating the effect of I & M delay times on equipment's deterioration and failure. In addition, this model integrates the deterioration of equipment's measurable conditions with effects of ageing on deterioration. The proposed model is solved using backward induction to obtain adaptive optimal policies with a less computational effort.

Using the solution of the proposed model, asset owners can perform inspection more cost effectively considering the knowledge about the current deterioration condition, the time that the equipment spent in that condition, the inspection delay time and the equipment's age. The solution also helps to perform maintenance more cost effectively considering the age, last known condition and the maintenance delay time. These adaptive policies are more useful, when maintenance has to be delayed in order to satisfy system requirements.

In a case study, we use CM and maintenance histories of transformers to demonstrate the model applicability. It is shown that the optimal CM actions vary with the equipment's condition, the time spent in the condition, CM delay time and the age of the equipment. The optimal maintenance actions vary with the equipment's condition, maintenance delay time and the equipment's age. This case study is conducted with data obtained from few transformers over an operational period of 7 years. Once the data is available over the extended period of time, the transition probabilities of different age ranges can be updated. Our algorithms can provide adaptive maintenance policies to other components in the system, given that the data is available. If required, the present value of money can be taken into consideration, when the model is solved using the backward induction method. We are currently applying this model to other components and to coordinate the maintenance activities of different equipment in the system.

Acknowledgement

The authors would like to thank the reviewers for constructive comments that help to improve the quality and readability of the paper. The authors would also like to thank local utility engineers for their comments and suggestions while building the model.

References

[1] R. Billinton, and M. Ran, "Composite system maintenance coordination in a deregulated environment," *IEEE Trans. Power Syst*., vol.20, no.1, pp. 485- 492, Feb. 2005.

[2] G. J. Anders, J. Endrenyi, G. Ford, and G. Stone, "A probabilistic model for evaluating the remaining life of electrical insulation in rotating machines", *IEEE Trans. Energy Convers.*, vol. 5, no. 4, pp. 761–767, Dec. 1990.

[3] P. Jirutitijaroen and C. Singh, "The effect of transformer maintenance parameters on reliability and cost: A probabilistic model," *Elect. Power Syst. Res*., vol. 72, no. 3, pp. 213–224, 2004.

[4] S. Natti, M. Kezunovic, and C. Singh, "Sensitivity analysis on the probabilistic maintenance model of circuit breaker", *9th Int. Conf. on Probabilistic Methods Applied to Power Systems*, Sweden, June 2006.

[5] M. Stopczyk, B. Sakowicz and G.J. Anders, "Application of a semi-Markov model and a simulated annealing algorithm for the selection of an optimal maintenance policy for power equipment", *Int. J. Reliab. Saf.*, vol. 2, no. 1/2, 2008.

[6] J. Endrenyi, G. Anders, and A. Leite da Silva, "Probabilistic evaluation of the effect of maintenance on reliability—An application," *IEEE Trans. Power Syst*., vol. 13, no. 2, pp. 576–582, May 1998.

[7] S. K. Abeygunawardane and P. Jirutitijaroen, "A realistic maintenance model based on a new state diagram", *11th Int. Conf. on Probabilistic Methods Applied to Power Systems*, Singapore, June 2010.

[8] S. K. Abeygunawardane and P. Jirutitijaroen, "New state diagrams for probabilistic maintenance models", *IEEE Trans. Power Syst*. vol.26, no.4, pp 2207-2213, Nov. 2011.

[9] Z. Xiang and E. Gockenbach, "Age dependent maintenance strategies of medium voltage circuit breakers and transformers", *Elect. Power Syst. Res*., vol. 81, no. 8, pp. 1709-1714, Aug. 2011.

[10] S. K. Abeygunawardane and P. Jirutitijaroen, "Effects of Maintenance on Reliability of Probabilistic Maintenance Models", *12th Int. Conf. on Probabilistic Methods Applied to Power Systems*, Turkey, June 2012.

[11] G. K. Chan and S. Asgarpoor, "Optimum maintenance policy with Markov processes", *Elect. Power Syst. Res*., vol. 76, no. 6-7, pp. 452-456, Apr. 2006.

[12] C. L. Tomasevicz and S. Asgarpoor, "Optimum maintenance policy using semi-Markov decision processes", *Elect. Power Syst. Res*., vol. 79, no. 9, pp. 1286-1291, Sep. 2009.

[13] S.V. Amari and L.H. Pham, "Cost-effective condition based maintenance using Markov decision processes", *Reliability and Maintainability Symp., RAMS'06. Annual*, pp. 464_469, 2006.

[14] H. Ge, C.L. Tomasevicz and S. Asgarpoor, "Optimum Maintenance Policy with Inspection by Semi-Markov Decision Processes", *39th North American Power Symp.*, pp. 541-546, Oct. 2007.

[15] F.I. Dehayem Nodem, J.P. Kenné and A. Gharbi, "Simultaneous control of production, repair/replacement and preventive maintenance of deteriorating manufacturing systems", *Int. J. Prod. Econ.,* vol. 134, no. 1, pp. 271-282, Nov. 2011.

[16] W. Ning, S. Shu-dong, L. Shu-min, S. Shu-bin, "Modeling and optimization of deteriorating equipment with predictive maintenance and inspection", *17th Int. Conf. on Industrial Engineering and Engineering Management*, pp. 942-946, Oct. 2010.

[17] W. Yan-ru and Z. Hong-shan, "Optimization maintenance of wind turbines using Markov decision processes," *Int. Conf. on Power System Technology, 2010* , pp.1-6, Oct. 2010.

[18] C. Dongyan and K. S. Trivedi, "Optimization for condition-based maintenance with semi-Markov decision process", *Reliab. Eng. Syst. Saf*., vol. 90, no. 1, pp. 25-29, Oct. 2005.

[19] M. J. Kim and V. Makis, "Optimal maintenance policy for a multi-state deteriorating system with two types of failures under general repair", *Comput. Ind. Eng*., vol. 57, no. 1, pp. 298-303, Aug. 2009.

[20] B. Castanier, A. Grall and C. Bérenguer, "A condition-based maintenance policy with non-periodic inspections for a two-unit series system", *Reliab. Eng. Syst. Saf.*, vol. 87, no. 1, pp. 109-120, Jan. 2005.

[21] M. S. Moustafa, E. Y. A. Maksoud and S. Sadek, "Optimal major and minimal maintenance policies for deteriorating systems", *Reliab. Eng. Syst. Saf.*, vol. 83, no. 3, pp 363-368, Mar. 2004.

[22] E. Byon, L. Ntaimo, and Y. Ding, "Optimal maintenance strategies for wind power systems under stochastic weather conditions", *IEEE Trans. Reliab.*, vol. 59, no. 2, pp. 393–404, June 2010.

[23] E. Byon, and Y. Ding, "Season-dependent condition-based maintenance for a wind turbine using a partially observed Markov decision process", *IEEE Trans. Power Syst.*, vol. 25, no. 4, pp. 1823–1834, Nov. 2010.

[24] M. Puterman, *Markov Decision Process*. New York: Wiley, 1994.
[25] A. E. B. Abu-Elanien and M. M. A. Salama, “Asset management techniques for transformers”, *Elect. Power Syst. Res*., vol.80, no. 4, pp. 456-464, Apr. 2010.
[26] Z. Lian, S. K. Abeygunawardane, and P. Jirutitijaroen, “Smart asset management of aging devices in energy systems: a case study of transformers”, *2nd European Conf. and Exhibition on Innovative Smart Grid Technologies*, Manchester, United Kingdom, Dec. 2011.
[27] IEEE Power & Energy Society, “IEEE Guide for the Interpretation of Gases Generated in Oil-Immersed Transformers”, 2008.

Saranga K. Abeygunawardane (S’07) is a PhD student at the Department of Electrical and Computer Engineering, National University of Singapore since July 2008. She received her B.Sc. (Eng) degree (Hon.) from University of Peradeniya, Sri Lanka, in 2007. Her current research interests include probabilistic methods, reliability theory and optimization techniques applied to power systems.

Panida Jirutitijaroen (S’05, M’08, SM'12) is an Assistant Professor at Department of Electrical and Computer Engineering, National University of Singapore. She received the B.Eng. degree (Hon.) from Chulalongkorn University, Bangkok, Thailand, in 2002, and the Ph.D. degree in Electrical Engineering at Texas A&M University in 2007. Her research interests are power system reliability and optimization.

Huan Xu received the B.Eng. degree in automation from Shanghai Jiaotong University, Shanghai, China in 1997, the M.Eng. degree in electrical engineering from the National University of Singapore in 2003, and the Ph.D. degree in electrical engineering from McGill University, Canada in 2009. From 2009 to 2010, he was a postdoctoral associate at The University of Texas at Austin.

Since 2011, he has been an assistant professor at the Department of Mechanical Engineering and Department of Mathematics (by courtesy) at the National University of Singapore. His research interests include statistics, machine learning, robust optimization, and planning and control, with applications in large scale systems.